Webinar – accompanying the Symposium “You want proof. I’ll give you proof.” – Mathematical, educational, & cultural perspectives

March 12th – 14th, 2025
University of Bremen

12th March 2025

08.45 – 10.05 *Keynote* Prof. Dr. Leander Kempen (University of Greifswald, Germany)

"And what do you see?" – The use of generic examples in the area of conjecturing, reasoning, and proof

Activities like conjecturing, reasoning, and proving are at the heart of mathematics. As such, several (inter)national standards emphasize their importance for mathematics education in schools. However, it has been shown that teaching and learning such activities at school and University levels is associated with significant problems. In recent decades, generic examples and generic proofs have been identified as valuable tools to engage learners in reasoning and proving. In my research group, we have extended the use of generic examples and integrated them into the conjecturing process of high school students to prepare and facilitate the transition to later reasoning activities. In my presentation, I will elaborate on the conceptual importance of generic examples and generic proofs, illustrate how to integrate generic examples in learning processes in conjecturing, reasoning, and proving, and highlight related findings from our empirical research.

10.30 – 11.50 *Keynote* Prof. Dr. Marcus Callies (University of Bremen, Germany)

A cognitive-cultural linguistic approach to the conceptualisation of PROOF and PROVING

Cognitive Linguistics, a school of linguistics that assumes a close connection between human socio-physical experience, mind and language, assumes that our conceptual system, in terms of which we both think and act, is fundamentally metaphorical in nature. Lakoff and Núñez (2000)[1] have argued that our understanding of basic mathematics is deeply linked to our experience of the world. They claim that we understand mathematics through conceptual metaphors that consist of mappings of experience derived from SOURCE DOMAINS (basic contexts and situations based on bodily or cultural experience, clear, simply structured, and concrete, e.g. spatial orientation) to TARGET DOMAINS (more abstract and complex contexts as often encountered in mathematics). In this talk I will discuss and exemplify metaphors and their metaphorical expressions used to conceptualise and talk about PROOF and PROVING in English. I will also explore potential cross-linguistic and cross-cultural differences in the conceptualisation of these concepts and possible implications for mathematics education.

[1]*Lakoff, G., & Núñez, R. (2000). Where mathematics comes from: How the embodied mind brings mathematics into being. New York: Basic Books.*

13th March 2025

08.30 – 10.15 *Project Presentation* Prof. Dr. Yusuke Shinno (Hiroshima University, Japan) & Prof. Dr. Takeshi Miyakawa (Waseda University, Japan)

International comparative research on proof and proving in school mathematics: some preliminary results and perspectives from a German-Japanese perspective

Proof and proving are seen as products and processes that are shaped differently in different countries. Since the start of the research project in 2020, we have been working on a theoretical framework, conceptualizing 'structure', 'language', and 'function' as principal aspects of proof and proving, and on methodological approaches to better understand cultural specificities of curricular documents and classrooms in relation to some of those aspects. This project presentation consists of several short presentations to introduce theoretical and methodological perspectives and to report on some preliminary results of comparative case studies between Germany and Japan in middle schools.

10.40 – 12.00 *Keynote* Prof. Dr. David A. Reid (Agder University, Norway)

The role of logic in an age of rhetoric: from proof to public discourse

Mathematical proof has been seen as a model for rational discourse for a very long time. Logic was an essential part of the mediaeval curriculum and Descartes kicked off the Enlightenment when he wrote "Those long chains composed of very simple and easy reasonings, which geometers customarily use to arrive at their most difficult demonstrations, had given me occasion to suppose that all the things which can fall under human knowledge are interconnected in the same way".[1] In a world facing very real dangers from climate change, war, disease and poverty, there has never been more of a need for public discourse based on reasoning, rather than rhetoric.
But what actually happens in mathematics classrooms? Is the approach to argumentation taken there suitable to prepare people to engage in reasoned social discourse? Is this goal even appropriate everywhere? In this talk I will explore these questions, and argue for some answers.

[1] *Discourse on Method* (1637). In The Philosophical Writings of Descartes (J. Cottingham, R. Stoothoff, D. Murdoch , trans). Cambridge University Press, 1985. Page 120

14th March 2025

09.00 – 10.20 *Keynote* Dr. Mika Gabel (Afeka Academic College of Engineering, Israel)

A rhetorical perspective on teaching proof at the tertiary level: Theory and practice

In this lecture, I will delve into the intersection of rhetoric and the teaching of proof at the tertiary level. I will start with a historical perspective on rhetoric and its relationship to mathematical proofs, particularly in teaching contexts, and then introduce 'The New Rhetoric,' the seminal argumentation theory developed by Chaim Perelman and Lucy Olbrechts-Tyteca with its key concepts scope and organization, basis of agreement, presence, and audience. I will propose how to adapt this theory to analyzing the teaching of proof and present analyses of lessons in Number Theory and Set Theory to demonstrate how this framework can be employed to examine real classroom scenarios via the notion 'flow of a proof'. I will highlight the formation of a shared "basis of agreement" between lecturers and students, identify gaps between their premises, and discuss the implications of such gaps.
My goal is to show that lecturers' mindful planning of rhetorical features of proof presentation can enhance classroom communication, help the lecturer to emphasize central elements of a proof, and reflect to the students the lecturer's mathematical values. I will show that intertwining "what" is taught with "how" it is presented facilitates lecturer-student dialogue and productive classroom communication.