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WHY SOME THEOREMS ARE NOT PROVEN IN GEOMETRY CLASS: DISPOSITIONS AND CONSTRAINTS

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Abstract: We research the work of the teacher in the context of proof and proving in secondary school geometry. In this paper we advance understanding of teaching phenomena that result in the fact that theorems are not always proven in US geometry classes, in spite of the fact that for a proposition to be theorem it has to have a proof. Our hypothesis is that the existence and nature of the proof of a theorem in a classroom is shaped by various dispositions that pertain to the rationality of teaching. We focus on those dispositions as they relate to the situation of installing theorems. Teachers' dispositions are elicited and analyzed in data obtained from 'thought experiments of mathematics teaching' among experienced teachers of US high school geometry. We argue that those dispositions can be organized according to a model of levels of teacher's activity and that this organization can help model the decision to prove or not to prove a particular theorem. The teacher makes a decision whether or not to prove a theorem by giving differential weight to different dispositions from different levels of activity.

Introduction

Mathematics education research on proof has explored the "potential" of proof in instruction. Much of this research on proof takes an epistemological or cognitive perspective, identifying different functions, roles, and types of proof (see De Villiers, 1990; Hanna, 1990, 2000; Balacheff, 1987; Harel & Sowder, 1998). We use here the adjective "potential," because not all the actual proofs found in real-world settings such as actual classrooms or in mathematical research necessarily play all those functions and roles (cf. Hanna, 1990; Rav, 1999). Our interest is to advance understanding of the nature of "real" proof and proving in the context of geometry teaching at secondary level.

In our previous studies (e.g., Herbst, 2002; Herbst & Nachlieli, 2007; Miyakawa & Herbst, 2007a), we have analyzed the nature of proof in geometry teaching from teachers' perspective in connection to different instructional situations.[2] The basic idea put forth by these studies is that the nature of proof might vary according to the instructional situation in which it exists. From observations of geometry lessons, we have found that some theorems are not proven when they are "installed" in spite of the fact that a proof might tell the class why the statement is true or what ideas that statement connects or requires. To explain why this happens, we hypothesize that it results from the fact that to manage one such situation teachers make use of a variety of dispositions that are part of their practical rationality and while some dispositions are resources to the work of teaching, other dispositions act as constraints. According to Bourdieu, practical reason or practical rationality is made up of the categories of perception and the categories of appreciation shared by members of a practice. The former involves "the categories available in a practice with which a teacher can identify and describe events or things," and the latter involves "the categories available in a practice with which a teacher can have an attitude toward, or allocate value to, events or things" (Herbst & Chazan, 2006). Those dispositions may help explain why the nature of proof is

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[2] We use the expression "instructional situation" to designate different kinds of activity that recur within a course of studies and that frame different exchanges (of work done for claims on what has been learned) between a teacher and her students. Two examples of instructional situations in US high school geometry are "doing proofs" and "installing a theorem" (see Herbst, 2006; Herbst & Miyakawa, 2008).

different in different situations. In this paper we address the question of how to account for the possibilities that a theorem is installed with no proof given or otherwise installed with a proof given. We produce that account by eliciting the different dispositions that might be at play for a teacher and show how these dispositions can play the role of conditions or constraints in the various kinds of work a teacher needs to do. Accordingly, different decisions as to whether to prove or not to prove a theorem might be accounted for by different combinations of these conditions and constraints.

Conditions and constraints on the nature of proof from an observer's perspective

From an observer's perspective, there are at least two kinds of conditions-and-constraints that shape the nature of proof; they do so at two different levels. On the one hand, there are constraints at the level of the shaping of the curriculum through the virtual debates among policy makers, textbook authors, and other stakeholders in which what to teach in high school is decided. In US high school geometry, some content specific to school mathematics such as the two-column proof form have resulted from such process: the two-column proof emerged as a result of a negotiation of multiple constraints— to justify the study of geometry on grounds of providing formal discipline and to allow a more diverse set of students to succeed in high school are two of those constraints (Herbst, 2002; González and Herbst, 2006).

On the other hand, when teachers decide what or how to teach mathematics in class they also make decisions in response to conditions and constraints. These demands are specific to the nature of classroom instruction. In terms of the theory of *didactic transposition* (Chevallard, 1991), the first set of conditions and constraints corresponds to the transposition from "scholarly knowledge" to "knowledge to be taught" and the second set to the transposition from "knowledge to be taught" to "taught knowledge."

In this paper, we are especially interested in the conditions and constraints of the second case; those constraints that teachers encounter when making decisions on how to transpose the knowledge to be taught into taught knowledge, in particular the decision whether or not to prove a theorem in class. From an observer's perspective those constraints might include things like required time, students' ideas, or available prior knowledge.

To account for how those constraints matter in teachers' decision-making, we need to inspect how elements of the practical rationality of teachers become constraints—how teachers' dispositions to see particular things or to value particular actions constrain the decisions they make. This paper contributes to earlier approaches to understand practical rationality in that it proposes to approach the question of organizing teachers' dispositions using a model of the multidimensionality of teacher's activity; the contribution of this model is that it helps theorize the role that dispositions play in activity and decision-making.

Levels of teacher's activity

A teacher's work consists of various activities, not all of them located in the classroom. For example, this work includes the construction of a syllabus or a description for a course of studies, the preparation of a lesson before teaching it, the tinkering with the lesson during teaching, as well as more general reflection on the general characteristics of their teaching and their students' learning. A teacher's activity is multidimensional.

Based on the theory of didactical situations (Brousseau, 1997) and in particular on Brousseau's model of the structure of the milieu, Margolinas (2002) has proposed a model that articulates a range of levels of a teacher's activity, to handle how the different dimensions of the teacher's activity might influence each other. In this model the complexity of a teacher's work is represented as a set of nested "games" a teacher (embodying various roles) plays against various milieus. The generative principle for all of those games is, as in Brousseau's original model, the notion that an agent interacts with and against an antagonist milieu that provides feedback on the agents' moves (see Margolinas et al., 2005). According to this model, the teacher's work can be understood as taking place at the following five different levels. (We complement the way Margolinas et al, 2005, describe these levels by describing how we understand each of the games the teacher is plays and against which milieu).

Level +3 Values and conceptions about learning and teaching
At this level the teacher is the possessor and possible a producer of an ideology of teaching or learning. The teacher plays a game that could be described as "enacting an ideology (of teaching)" against a "construction" milieu that could include things like an vaguely described area of mathematical knowledge, a vague description of students, a long or undefined period of time, and coarse description of behaviors and materials (e.g., group work, standing in front of class, technology, manipulatives). An example of what a move by the teacher might be like in this game is a statement like "it is important for a teacher to promote students' activity with concrete materials" or "learning disabled students need a lot of repetition."
Level +2 The course of instruction seen globally
At this level the teacher is the constructor of a course of studies who makes global and thematic decisions regarding the course she teaches, playing this game against a milieu that includes an itemized but disorganized list of mathematical ideas to be taught, a school-curriculum-related description of students (they have studied this or that topic in prior years), a bounded but long period of time (the year, semester, week), a relatively stable inventory of activity structures and goals that those could serve, etc. An example of what a move by the teacher might be like in this game is a statement like "we give the windmill proof of the Pythagorean theorem as a second proof of this theorem."
Level +1 The local course of instruction didactic project (or the lesson)
At this level the teacher is the planner of a lesson who makes local and sequential decisions regarding the way one particular theme of the curriculum is taught, playing this game against a milieu that includes an itemized and sequenced list of mathematical ideas to be taught, a course-curriculum-related description of students (they have seen that topic last week, half of them are failing), a bounded and short period of time (the day, the hour), a specific set of objectives and possible activities that could be used to meet those, etc. An example of what a move by the teacher might be like in this game is a statement like "to help them understand that slope is a constant I will give them the coordinates of several points on a line and ask them to pick two and calculate 'rise over run,' then share what each got, that will surely surprise them."
Level 0 Instructional action
At this level the teacher is so in the strict sense of being in the class assigning work, talking to students, and managing discourse among students. He or she plays this game against a milieu that includes identified students' misconceptions, expected student responses, individual students, etc. An example of what a move by the teacher might be like in this game is a statement like "let me give you an example."
Level –1 Perception of pupils' activity
At this level the teacher is an observer of students' work and someone who sustains the students' milieu without uncovering its didactical intentions. As observer, the teacher attends to emerging solutions to problems by students and matches them with categories of observation. An example of what a move by the teacher might be like in this game is the following note made by the teacher in his notebook like "Matthew forgot to invert before multiplying."

These levels are proposed to account for the multidimensionality and connectedness of a teacher's work. Each dimension is defined by the activity the teacher is involved in and the milieu against which the teacher needs to act, while the specific actions she takes may satisfy one or more of these activities. As noted by Margolinas, the model does not provide a linear, temporal description of a teacher's activities—it should not be inferred that a teacher always makes ideological decisions first, then curricular ones, then lesson-planning ones, then tactical ones, and eventually observes students. The model does not account for the location of those moves either—it should not be inferred that while a teacher teaches she only makes moves at level 0. The model allows us to organize different moves and decisions that are part of a teacher's activity (regardless of their visibility) on the basis of the kind of agent and the kind of milieu that these moves and decisions target.

The model of structure of milieu contains the hypothesis that the activities a teacher engages in are connected and this connectedness is reflected in the what the milieu the teacher interacts with at each level consists of (cf. Perrin-Glorian, 1999; Margolinas, 2002). On the one hand, the milieu includes a component that consists of the results of activity of higher levels. We use the notation "Msup+n" to denote this component of milieu. On the other hand, the milieu includes a component that consists of the results of activity of lower levels. We use the notation "Minf+n" to denote this component. To model what the milieu is at any given level we use the heuristic that the milieu at each level (+n) is composed of the aggregation of Msup+n and Minf+n. For example, when making a decision about the plan of a specific lesson (level +1) (e.g., whether to give an example of the Pythagorean theorem before or after the proof of the theorem and how to develop this example), this decision contends with and against the result of a thematic decision (about what objects to teach) made at level +2 (e.g., whether the Pythagorean theorem is a proposition about areas to be proved using the windmill argument or a proposition about lengths to be proved by similarity). Thus the outcome of this thematic decision is part of Msup+1. At the same time, the planning decision (e.g., when to give an example and how to develop it) made at level +1 is hypothesized to require consideration of the possible realizations of instruction (level 0) and of those students' action that are anticipated or remembered by the teacher (e.g., "because last year students could not connect the statement about areas in the Pythagorean theorem to problems of finding sides in a triangle, this year I will exemplify how the theorem allows one to calculate a side"). Thus, the outcome of these instructional processes is part of Minf+1. In general, the nested nature of the different levels of activity entails that actions and decisions in the making at one level are products that constitute the milieu for activity at another level.

We use this model of teacher's activity as a tool to analyze and organize how the dispositions of practical rationality (particularly dispositions to see or value certain things at one level) can act as constraints for the teacher's activity at another level. We apply this idea to the modeling of a teacher's decision making whether or not to prove a theorem.

Research questions and mode of inquiry

Research questions

In previous reports of the study of teachers' practical rationality, we have identified dispositions toward proving theorems that might have an impact on whether a particular theorem is proved or not. In Miyakawa & Herbst (2007a), we reported that there is evidence that for teachers not all theorems need to be proved. Some of the reasons they provide allude to time constraints, to what is important about the theorem for students (discovering it, using it, or being able to prove it), and to anticipated or observed students' difficulties understanding a proof. In Miyakawa & Herbst (2007b), we showed that the existence of a discontinuity between the ideas that allow students to corroborate the truth of a statement and the ideas used in proving the statement could be one reason why not to prove a theorem. Our present question is how these dispositions can be organized so as to use them more precisely to justify or anticipate decisions. While we know that some dispositions may enable a teacher to decide whether to prove a theorem, our theoretical framework is as of yet underdeveloped in regard to allowing us to decide how different dispositions relate to each other so as to precipitate a particular move or decision. In this paper we investigate how the model of teacher's activity levels described in the prior section can help fulfill those purposes. We substantiate the conjecture that these dispositions act as conditions and constraints on a teacher's action and decision-making.

Data

The data to be analyzed was gathered in study group meetings among experienced geometry teachers who were engaged in 'thought experiments in mathematics teaching' (Herbst & Chazan, 2006). In these thought experiments, teachers confront together representations of teaching: stories of classroom interaction represented as animations of cartoon characters. Some of those stories show different ways in which theorems could be installed. Teachers discuss these stories: In particular, they flag moments when they might have done otherwise and indicate what they might have done instead. The data consists of discussions on the represented teaching and on the possibilities that teachers conceive in response. In this paper, we use data gathered from a session in which an animated movie was shown. In the animation (called "Intersection of medians") the teacher proves a theorem about the centroid of a triangle with little input from students who appear not to follow well the proof (see Miyakawa & Herbst, 2007a).

The discussions among teachers in the study group meetings have been parsed into small conversations that we call intervals and which are delimited by practitioner-generated conversation boundaries. Each interval has been analyzed using linguistic tools in such a way as to extract the participants' interpretations of the stories they are watching as well as the alternative stories that they envision could happen. In any one interval it is customary to find an array of stories related to a common animation, often branching off common moments. Participants customarily use elements of the appraisal system of language (Martin and White, 2007) to appraise those stories or moments in those stories as far as their comprehensibility, desirability, importance, normativity, probability, seriousness, or usuality (Lemke, 1998). We model those appraisals as arguments using Toulmin's argumentation model, which enables us to relate reported or proposed story elements to more general statements of fact or principle provided in support for one or another story. All of those statements that either report or propose actions in stories or else justify those are empirical material that fleshes out the dispositions of practical rationality. We use the model of teacher's work (as nested games against a milieu) presented above to organize the various elements of the arguments that participants advance during the study group discussions, locating their place in the model as if they were products of a teacher's work done at one of the different levels of activity. For example, the watching of an animation may prompt a participant to make an interpretation about students' thinking and this would be classified as a statement made at level -1. Also, when justifying a teacher's decision to showing students a proof a participant might argue for the value of giving students the chance to observe expert mathematical performance, which would be classified as a statement of level +3. As noted by Herbst & Chazan (2006), our animations often immerse participants in the action to the point that they impersonate the teacher in the animation, talking as if they were that teacher interacting with their students. We classify those statements as actions at level 0 (actual teaching). This paper contends with a mass of statements obtained from two intervals.

Conditions and constraints on the decision whether or not to prove a theorem

We present here some of the analysis of data in which US high school geometry teachers discuss whether or not to prove a theorem. The discussion among participants gives us material to identify dispositions that can be classified in different levels. And we show how the teachers' dispositions activated for a given activity play a role of conditions and constraints in another activity. This approach allows us to model how different dispositions interact with each other in the process of warranting practical actions or decisions. It does that in the sense that the dispositions found and the model of levels of activity help reconstruct the milieu of the teacher and predict decisions made.

What leads to the decision that the theorem will not be proved

Stories and decisions

The data we deal with can be described in general by saying that practitioners respond to the stories of instruction showed in an animation by narrating other stories—either stories of what they see in the animation or stories of what they might or might not do instead in similar circumstances. In our analysis of intervals we distill the stories that practitioners narrate and identify dispositions as warrants for story moments. In one of the intervals (Interval 23 of Session 121505) containing discussions of "Intersections of Medians" we have identified two stories related to whether or not to prove the theorem given in the animation. One of those stories is the participants' narration of the animated episode. The other story is an alternative to the first one. These stories are represented in the following excerpt of transcript:

Moderator	They still were what?
Denise	Confused! I mean, that could have been - I don't know. I think him [the teacher] explaining it made it worse. If he would have just stated the theorem, had a brief discussion about it, it would have been ok. Him trying to prove it and prove it so fast, and trying to keep the class to keep up with him, instead of following the lead of the class--huh?

The interpreted story can be described as including the following three moments.

1. The teacher proves the theorem very fast
2. The class has to keep up with the teacher's exposition
3. Students are confused

Denise summarized this story as "trying to prove the theorem made the teaching worse." One can identify a negative appraisal here—"worse" points to the undesirability of such teaching characterized by the proving of the theorem. Instead of this story, the participants proposed the following alternative story in which the theorem is not proved. The story consists of the two following moments.

1. Teacher just states the theorem
2. The class has a brief discussion about the theorem

The elements of the second story (stating and discussing the theorem) show actions that a teacher would do in actual classroom instruction—they happen at level 0 of the model. At the same time, we can also identify here (as part of Msup0) a decision made to not give a proof (by the use of "just"), which allows stating to be followed by discussing. This decision is a result of teacher's activity at level +1 where the teacher's goal would be to plan how the theorem is dealt with in a lesson. In this paragraph, we are going to analyze the dispositions around the decision "no proof" and show how we can model the teacher's decision in terms of the milieu.

Organization of dispositions for the decision to provide "no proof"

In order to produce a decision "no proof" as a result of activity at level +1, according to the structure of milieu, two components of milieu need to be considered: Msup+1 and Minf+1. An element of Minf+1 has been identified above as the perception that "trying to prove the theorem made the teaching worse." This is a reason Denise explicitly stated; it is based on her observation of animated instruction as described in the interpreted story.

As a result of identifying that perception as part of M+1, we can infer other dispositions that belong to M+1. The evidence for that inference are the decisions made to teach the theorem, to not prove it, and the perception identified that proving would make it worse. The decision to teach this theorem is part of M+1 and cancels the alternative of possibly not teaching this theorem. That decision to teach this theorem results from activity at level +2 where the teacher constructs the scope and sequence of themes of the course. That activity would produce the decision "this theorem needs to be taught." This is one of elements of the component Msup+1.

The warrant that supports the decision “no proof” is the perception that “proving the theorem made the teaching worse.” The negative appraisal expressed in this perception allows us to infer an underlying disposition for this decision: “bad teaching should be avoided.” In fact, if teaching were not perceived as amenable to such judgment (bad/good) or if such judgment was not perceived as avoidable, there would be no reason to negatively appraise it and change the teaching. This disposition toward judging teaching as good or bad is a general conception about teaching, produced in the teacher’s activity at level +3. It becomes an element of the component Msup+1.

The decision “no proof” cancels not only the alternative of proving this theorem as shown in the animation, but also the alternative of proving this theorem in another way. If all at stake was to avoid “bad teaching” this might be accomplished, for example, by proving the theorem more slowly, by using another proof, etc. In spite of these other choices, the participants chose “no proof.” There should be a reason that leads to this decision. A disposition we can propose to enable such inference is that the role of proof is undervalued in teaching: A proof is not always needed to install a theorem. This disposition could be located at the +2 level of global planning of instruction: theorems and their proofs are seen as separable, unlike 150 years ago when versions of Euclid’s Elements were used to teach geometry (see Herbst, 2002).

In this way, one may model the decision “no proof” with a milieu that consists of dispositions produced at different levels of teacher’s activity. We argue that these dispositions play a role of conditions and constraints for the decision not to prove the theorem at level +1. The following diagram (Fig. 1) summarizes this modeling.

T+1 (planner) <>	Msup+1: *Bad teaching should be avoided (+3)* *A proof is not always needed to install a theorem (+2)* *This theorem needs to be taught (+2)*
Decision: no proof	Minf+1: Proving this theorem made the teaching worse

Figure 1: Activity for the decision “no proof” [3]

This diagram is focused on the decision “no proof” that we are interested in. This does not show the process of production of each element or the interactions between teacher and milieu at other levels that might have happened beforehand (some might have happened some time ago, and others might have happened when the participants watched the animated movie).

What leads to the decision that the theorem will be proved

Stories and decisions

In another interval (Interval 25) of the same session of the study group (Session 121505), we have identified another story and dispositions that support to prove the same theorem. The following excerpt of transcript describes them.

Megan — I think that particular theorem, it’s not very useful on its own after that. But the proof is actually very useful. I think that's a perfect example of a theorem where the proof's a lot more useful than the theorem is, in the end. Because you're talking to kids about the area of a triangle, and what is the crucial thing to know? I need to know the base and the height, and the fact that the—you have different triangles that have the same base and height and they all have the same area – I don't know, I think that's an example of where the proof is. The guy went way too fast. I agree with that. But if you spend a long time on that, the proof's more useful--afterwards you never use that

[3] “T+n” denotes in this paper the teacher at level +n. Dispositions in italic are those hypothesized.

the areas are equal. They don't use that. Do you ever use that? No.

The proposed alternative can be described as including the two following moments:

1. Teacher spends more time showing the same proof
2. Teacher talks about area of triangles, bases and heights, and equal area

These story elements describe the actions that a teacher might do in an actual lesson (level 0). The actions toward completing the first moment (prove slowly) might overlap with the actions to accomplish the second moment (talk about some concepts). We are here interested in the decision to "prove the theorem" at level +1 that would be fulfilled as the above story unfolds. We model the emergence of this decision in what follows.

Organization of dispositions for "provide this proof for the theorem"

We can identify the main warrant that supports to prove the theorem in the above excerpt of transcripts:

- The proof is very useful

This disposition is also a warrant for a thematic decision "teach this theorem" or a disposition "this theorem needs to taught" which can be identified in the given story and which is also one of elements of the component of Msup+1 (in the similar way as the decision of "no proof" presented above). We consider that the usefulness of the proof is a disposition of the proof that is produced in the activity of level +2 where a thematic decision is made. This disposition itself is not a thematic decision of what to teach, rather a product of interaction between the teacher and the proof during the activity in which a thematic decision is made (teach this theorem). Therefore it belongs to the component of Msup+1.

The usefulness of the particular proof considered would be an influential element for deciding to prove the theorem. However, the sole disposition to value this proof as useful would not force a decision to prove this theorem. Like in the alternative story (no proof) proposed above, if no student in the classroom could understand this proof, the theorem might still not be proved (and perhaps the teacher might not even teach the theorem), even if the proof was useful as a mathematical container for a technique or strategy. What allows a teacher to decide to provide this proof for the theorem will be the disposition that it is possible for "the teacher [to] spend a long[er] time on the proof." This disposition belongs to the component of Minf+1, since when experiencing the way the proof was presented in the animation, participants could notice that the showing of the proof took just a little over five minutes.

Thus, the dispositions around the decision to "prove this theorem" can be organized as in Figure 2.

<table>
<tr><td rowspan="2">T+1 (planner) <>

Decision: prove theorem</td><td>Msup+1:
This proof is very useful (+2)
This theorem needs to be taught (+2)</td></tr>
<tr><td>Minf+1:
It is possible for a teacher to spends longer time on this proof (0)</td></tr>
</table>

Figure 2: Activity for the decision "prove this theorem"

Since the crucial disposition in this decision is the usefulness of the proof, we try to model here the process of its production in the activity of level +2. The decision made at this level (the proof is useful) is based on several dispositions. One can identify one of dispositions in the story elements presented above: The teacher can talk about some key concepts (area of triangle, equal area, and base and height). This disposition expresses a reason why the proof is useful.

In order to infer other dispositions that might matter in coming to the conclusion that the proof is useful, one could ask why is a proof useful if some concepts can be mentioned. On the one hand, this usefulness might depend on the nature of concepts. If the concepts mentioned are out of reach for students, or contrary if the concepts mentioned are completely mastered or too basic (e.g., what students learn at the first grade of primary school), the proof might not be useful. So, we hypothesize here a disposition that *the concepts at play in the proof are related to what students have already studied but still at stake for them*. In other words, these concepts are still objects of teaching for the target class. This disposition might belong to the component Minf+2 since it might originate from activity at level +1where a teacher would be planning specific opportunities for her students to revisit concepts that are still at stake. The disposition could also be produced in activity at the level +2, for example as a result of the interaction of the teacher contending with the need to make thematic decisions about the teaching of area in the context of a deductive geometry course and against the disposition issued from Minf+2 that students have already encountered area as calculation in earlier courses. Indeed the sense to which it is important for students to learn to make claims about area without calculating areas was an explicit element of the negotiation of the experimental contract in the instructional experiment reported by Herbst (2006), in which Megan Keating (the same teacher who participated in this study group meeting) engaged her students in developing a proof that is quite similar to the one presented by the teacher in the animation shown (see also Herbst, 2005).

Discussion and Conclusion

We have reconstructed the decision making process behind two possible stories, presented by teachers as alternatives to the animation “Intersection of medians”: In one of those stories the theorem is not proved while in the other one the same proof of the theorem is given. We have done that by organizing teacher dispositions elicited from the analysis of study group data—especially the warrants that teachers provided for particular moments in stories. We have used the model of different levels of teacher’s activity to organize teachers’ dispositions in the different levels where they could be operational. We have also used the model to make hypothesis of other dispositions that when added to those elicited from the data would allow the model to produce the decision targeted.

As these dispositions (observed and hypothesized) account for (or force) a particular decision we consider that these dispositions play a role of conditions (resources) and constraints in a teacher’s work that is to say: a decision is made with the assistance of those dispositions. To see how the two cases examined above can help further research on teaching, suppose that a theorem has to be taught and a teacher has to decide whether to prove it. Based on the two cases discussed above, one could model the process of decision as including smaller decisions or questions that the teacher would have to ask, as follows. A first question would be to anticipate how much the giving of a proof might negatively affect the quality of the instruction (would it make the teaching worse?). A second question is whether the proof at hand is useful to accomplish other thematic goals of teaching. An anticipation of whether students might be confused by the proof at hand would be a criterion to answer the first question. A consideration of what concepts are used in the proof at hand and whether these concepts are objects of study would help answer the second question. The decision whether to give the proof of the theorem can be modeled as an outcome of a process that uses the information generated through those questions.

Of course, the dispositions identified in the data are only illustrative of the warrants that teachers provide to justify alternative courses of action, other dispositions might be found in comparable responses to an animation like “Intersection of medians.” Further accounting of empirical data, using the model of levels of activity, could therefore add features to the model proposed for the decision of whether to give that proof to that theorem. Teachers’ responses to other animations where a proof for a theorem is done or is missing can similarly help account in more robust way for the decision of whether to prove or not to prove a theorem. In our data, we have identified many other criteria (e.g., whether the proof requires triangle congruency, whether the proof is beautiful, whether students request a proof, etc.). By incorporating those elements to a model of the levels of teacher activity in the way we have illustrated above, we expect to advance toward a more complete model of how the decision whether or not to prove a theorem is produced and how those different dispositions become conditions and constraints to that process.

As the two cases illustrate, the dispositions involved might not be active at the same time or with the same strength. In one interval, the participants were disposed to consider the mathematics of the proof while in the other they were disposed to consider the potential confusion of the students. The extent to which the participants would be aware of both considerations when planning is unclear, yet it seems reasonable to expect that even if they were so aware, the way they might weigh each of them might vary (Herbst & Chazan, 2003). A reasonable outcome of the work of modeling decision making in teaching is not a deterministic model that would predict every single instance of action, but rather a model that accounts for all the dispositions that could become possible conditions and constraints for a given action and all possible arrangements of those conditions and constraints into different, possible paths for action.

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