

# DEVELOPING MATHEMATICS TEACHER KNOWLEDGE: THE PARADIDACTIC INFRASTRUCTURE OF “OPEN LESSON” IN JAPAN\*

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## **Abstract.**

In this paper, we first present a theoretical approach to study mathematics teacher knowledge and the conditions for developing it, which is firmly rooted in a systemic approach to didactic phenomena at large, namely the anthropological theory of the didactic. Then a case of open lesson is presented and analysed, using this theoretical approach, to show how the format of open lesson contributes to the construction and diffusion of didactic knowledge in the community of mathematics teachers in Japan. The basic idea of this format is that teachers from other schools are invited to observe a class taught by a teacher then participate in a discussion session with him on the details of the lesson. For the case study, we analyse the lesson plan prepared for an open lesson, the observed lesson, and the teachers' discussion. As a result, an open lesson session has been described as a specific form of post-didactic practice related directly to an actual observed lesson, and aiming specifically at elaborating the theoretical aspects of teacher's didactic practice in the lesson and beyond it.

## **Key words.**

mathematics teacher knowledge; teachers' professional development; open lesson; Japan; elementary school; Anthropological theory of the didactic.

## **Introduction**

Mathematics education researchers have recently given substantial and increasing attention to the knowledge and practices of mathematics teachers (for some early syntheses, see Bromme, 1994; Chevallard, 1997; Ball, 1997; Jaworski, 1998). In particular, the ways in which this knowledge and practice *develops* – or could be developed – is central to the rapidly expanding literature on prospective and practicing mathematics teacher education that may in fact be ascribed to this theme of research (for a synthesis based on the 15<sup>th</sup> ICMI study, see Even & Ball, Eds., 2009). As regards to practicing teachers, the research domain is not limited to the study of organised “educational” activities for teachers (such as workshops, courses, etc.). It also encompasses spontaneous and autonomous forms of teacher professional development, whether on an individual or collective basis.

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It is an important research task to model and analyse teachers' learning from practice. As regards collective forms of autonomous teacher development, the Japanese format of *lesson study* is rather well described in the literature (e.g. Isoda et al., 2007; Fernandez & Yoshida, 2004). In this paper, we present and analyse another common Japanese form of professional development of experienced teachers: *open lesson* (in Japanese, *koukai-jugyou*) which occurs, for example, in the setting of teacher *study meetings* (in Japanese, *kenkyu-kai*). This kind of lessons is also called *public research lesson* in the USA (see Lewis, 2000; Lewis et al., 2006). The basic idea of this format is that teachers from other schools (up to several hundred) are invited to observe a class, taught by a teacher of the school, and just after this, to participate in a discussion session with the teacher – and sometimes other invited experts – on the details of the lesson. As we shall see, *open lessons* present both significant similarities and differences with respect to *lesson study*.

The aim of the research presented in this article is twofold:

- (1) Present a theoretical approach to study mathematics teacher knowledge and the conditions for developing it in direct relation to teaching practice. Our model is new, but it is firmly rooted in a well-established systemic approach to didactic phenomena at large, namely the anthropological theory of the didactic (introduced in the section “Theoretical model”);
- (2) Present and analyse a case of *open lesson* (observed during an open school festival in Joetsu in 2009) using this theoretical approach, and thereby demonstrate in a detailed and systematic way how the format of *open lesson* can contribute to the construction and diffusion of didactic knowledge in a community of mathematics teachers.

The article is structured as follows: we first develop the research context for the present study to situate it in the literature on teachers' professional development. Then we present our theoretical model, the context and methods of our case study, and the findings from that study, based on our theoretical model. Finally, in the last section, we discuss the wider didactic perspectives of the format of *open lesson* and our theoretical analysis of it.

## Background

A seminal paper by Margolinas, Coulange and Bessot (2005) poses by its title what is, in a more special sense, the basic quandary also of the present research: “What can the teacher learn in the classroom<sup>1</sup>”? These authors develop the notion of *observational didactical knowledge* which “grows from the teacher's observation and reflection upon students' mathematical activity in the classroom” (ibid., p. 205). The fact that they talk here of a single teacher is significant and corresponds both to

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<sup>1</sup> The *classroom* is here to be construed as the site of the teacher's central activity, that of teaching in the sense of supporting students' learning through direct interaction with them.

the data they analyse and to the common situation of many teachers: they are *alone* as teachers in the preparation and conduct of their teaching, and consequently a significant part of their observational didactical knowledge remains private. This leads to a slightly modified form of the above question: “What can teachers learn together, inside and outside the classroom?”

We notice that the growing international interest in Japanese lesson study (cf. Hart et al., 2011) is linked to its deliberate methods to promote a *shared* approach to teachers’ observation-based learning from teaching, as well as a sustained and goal oriented long term collaboration beyond the classroom. More generally, the notion of “professional learning community” (cf. McLaughlin and Talbert, 2006) has become a main object of research and strategic intervention related to teacher development. There are well documented projects in many countries which focus on experimental teaching and which involve long term collaborations between mathematics education researchers and teachers (e.g. Jaworski, 1998). In most of these studies, the question of *sustainability* (cf. Richmond and Manokore, 2011) of teachers’ learning communities emerge explicitly, and it leads to an interest in wider institutional and cultural conditions which support, or hinder, such communities to flourish and last beyond experimental and privileged settings.

A related question is that of the *resources* on which teachers may draw in collective or individual efforts to develop their teaching (cf. Gueudet and Trouche, 2009; Winsløw, 2011) – for instance, teacher networks organised in technology-based platforms may enable collective development of resources across institutions and even countries. These modalities of sharing and collectively developing resources appear indeed to be of increasing importance to many teachers, but it is also clear that it does not lead to shared didactical observational knowledge based on common experiences of real time teaching. Nevertheless, the centrality of material products of the common experience – *documents* in the sense of Gueudet and Trouche (2009) – points to the question of how to share those documents beyond the group directly involved in the teaching or observation of it (cf. also Winsløw, 2011). At the same time, we face the question about conditions that permit the sharing of observational didactical knowledge across and beyond individual schools, if that knowledge should really be conceived of as knowledge linked to the profession (rather than to a particular group or school, cf. e.g. Stigler and Hiebert, 1999; Cirade and Chevallard, 2010). Both questions are illuminated by the theoretical model and by the case presented in this paper.

### **Theoretical model**

Models are tools to identify and describe a part of reality, and they are usually anchored in a more general theoretical framework. Our model is not specific to the study of open lessons, but helps to situate this format in the wider context of “mathematics teachers’ practice and knowledge related to teaching”, as identified in the previous section. The model is based on a well established theory from the didactics of mathematics, which we outline before describing the model itself.

### *The anthropological theory of the didactic (ATD)*

The original formulation of the theory is due to Chevallard (1997, 1999, 2002), cf. also Barbé et al. (2005) for a more extensive overview; we now briefly present the elements we shall need here.

Mathematics teaching and learning situations are characterised by the construction and sharing of *practice* and *knowledge* of a mathematical kind. An organisation of practice and knowledge is called a *mathematical organisation* (MO) and as we shall see, the theory provides a refined framework to specify the components of a MO. In common parlance, MOs are often identified and declared in broad terms like “arithmetic”, “quadratics” or “linear regression”. Such terms can be found in curricula and text books, but they do not suffice to describe the practice and knowledge enacted in a concrete situation (like a lesson, homework, etc.). Also, the teacher will have to develop a specific kind of practice and knowledge, roughly “that of teaching the MO”; such organisations of practice and knowledge are called *didactic organisations* (DO) in ATD. We note that a MO developed in the classroom *depends on* the DO, and in particular on what the teacher does and knows; and, conversely, the DO is developed accordingly to comply with certain aims for the MO, as well as with other restrictions in terms of pedagogy, time, material conditions, etc. In ATD, this interdependence of DO and MO, and the common dependence on exterior constraints, is described as a *co-determination* of DO and MO which are then the principal object of research, detailing the variety of conditions, resources and constraints which determine what we see in the situation (and analyse in terms of a DO and a MO).

In ATD, an organisation (didactic or mathematical) is viewed as a family of *praxeologies*, which are elementary units of practice and knowledge. These units are defined by a single *type of task*  $T$ , solved by a *technique*  $\tau$  (the means to solve all tasks of a given type); any *practice* is modelled as being related to such a pair  $[T/\tau]$  (practical block of praxeologies). But human techniques may be more than simply “done”, they can also be explained, which means that they can have what ATD calls a *technology*  $\theta$  (a discourse for the technique). Finally, technologies may themselves be explained and justified, an abstractive level which ATD terms *theory*  $\Theta$ . The knowledge related to the practice is therefore modelled with a pair  $[\theta/\Theta]$  (theoretical block of praxeologies). In short, ATD models any human practice in terms of praxeologies, which are quadruples of the form  $[T/\tau/\theta/\Theta]$  (type of task, technique; technology, theory).

The fact that praxeologies are often occurring in coherent “families” is reflected in the name *organisation*. For instance, a MO which we could loosely call “subtraction”, might be unified by a technology about how to do subtraction in different cases (types of tasks). In this case, a DO to teach “subtraction” departs from the task of identifying concrete tasks for students to work with, and to arrange and support this work so that it develops an “appropriate” MO. The techniques and technologies of the DO might also be explained and justified by the teacher, thus developing a

“didactic theory”. Such a theory may be of a very personal or institutional nature, but it may also be strongly related (or appealing) to official documents and other “theories” shared across institution.

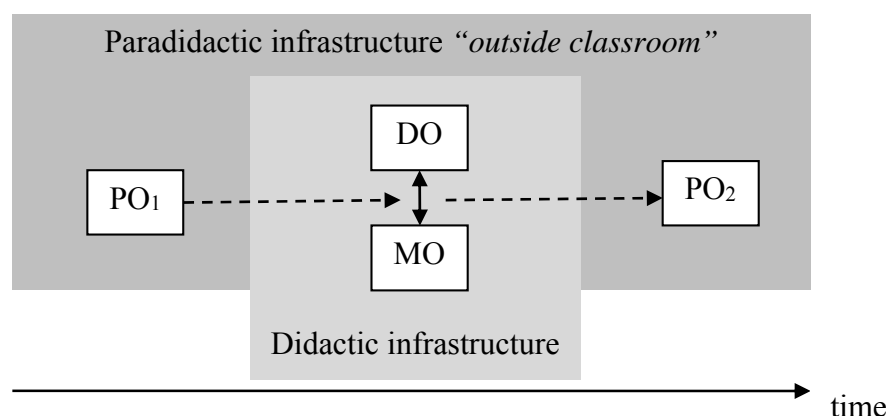
*Our model: Mathematics teacher knowledge and paradidactic infrastructures*

Mathematics teacher knowledge obviously includes technological and theoretical elements of MOs and DOs, and some of the theoretical elements may be quite general (e.g. “pedagogic”) and be widely shared within the school institution. Notice that “knowledge” is used here in a broad sense, which includes individual “knowings”; however, in our study we shall focus on knowledge shared in an institution and comprising in particular a shared technology.

As shown by Winsløw (2011), the *conditions* for shared practices – inside and outside the classroom – can take different forms and these differences may have crucial effects on the degree to which teachers develop shared and warranted knowledge about the co-construction of MO and DO. The set of conditions for the teachers’ work in the classroom – with a concrete MO and DO – are called a *didactic infrastructure* (cf. Chevallard, 2009), and refers ultimately to the levels of co-determination of those organisations, particular to the school institution, its pedagogy, the discipline and so on.

The set of conditions for the work *outside* the classroom, related to a given MO and DO, is called their *paradidactic infrastructure* (Winsløw, 2011). For instance, the teaching of new parts of the curriculum may give rise to more intensive collaboration of teachers, with special resources available. And whether or not they support shared work of teachers outside classrooms, these paradidactic infrastructures contribute to frame the teachers’ development of knowledge *about* the co-determination of MOs and DO. As in the case of teacher knowledge *within* these organisations, the knowledge *about them* – and in particular about their co-determination – is more easily studied, and perhaps also more systematically developed by the teachers themselves, in those cases where the paradidactic infrastructure favours collaborative work *outside the classroom*. This work, in general, corresponds to *paradidactic organisations* (PO) of teacher practice and knowledge. It is useful to distinguish two parts of it (corresponding to a split in time): practice and knowledge developed *before* (PO<sub>1</sub>) and *after* (PO<sub>2</sub>) the teaching practice itself, as shown in Fig. 1. These *pre-didactic* and *post-didactic organisations* include the practice and knowledge involved in *preparing* and *evaluating* the co-construction of MO and DO.

We notice the possibility of a developing paradidactic organisations *simultaneously* with the realisation of MO and DO, when some of the teachers observe the teaching (as in Japanese lesson study, which can be considered a specific set of paradidactic conditions, cf. e.g. Winsløw, 2011). However, the setting of paradidactic organisations *before* and *after* the co-construction of MO and DO represent the minimal and typical case of teaching practice.



**Fig. 1. Paradidactic and didactic infrastructure conditioning teachers' common development of practice and knowledge *inside* and *outside* the "classroom" (teaching situations). See Table 1 for the abbreviations used.**

Abbreviation	Meaning
<b>MO</b>	Mathematical Organisation: a collection of mathematical praxeologies (e.g. those developed in a lesson, a course, etc.)
<b>DO</b>	Didactical Organisation: a collection of didactic praxeologies (e.g. those used to implement a lesson, a course, etc.)
<b>PO<sub>1</sub></b>	Pre-didactic Organisation: a collection of praxeologies developed to <i>prepare</i> a teaching activity (e.g. those enacted by a teacher to prepare a given lesson)
<b>PO<sub>2</sub></b>	Post-didactic Organisation: a collection of praxeologies developed to <i>evaluate</i> and <i>revise</i> a teaching activity
<b>PO</b>	Paradidactic Organisation, consisting of PO <sub>1</sub> and PO <sub>2</sub> corresponding to the same teaching activity

**Table 1. Central notions and abbreviations used throughout this paper.**

## Context and Methods

### *Open lesson as an element of paradidactic infrastructure*

In many prospective and practicing teacher education activities, participants read and talk about abstract DOs and MOs, but without shared and direct access to any concrete instance of a lesson (with a realised DO and MO). Even when practice periods are included in prospective teacher education, it is often a central difficulty to relate the experience from practice with the contents of the education at large. When it comes to practicing teacher professional development, this difficulty may be even

greater, and it is the aim of this paper to analyse and present a concrete instance of an approach to this difficulty which is very common in Japan<sup>2</sup>.

This format is called *koukai-jugyou* (“open lesson”). Its most apparent features are very simple: a large number of teachers observe a lesson taught by a colleague and discuss various aspects of it right after the lesson. Besides teachers, there may also be other guests, such as university researchers, who participate in the observation of the lesson and in the discussion afterwards. The only other significant feature of the DO of the lesson is an explicit theoretical block, made available to all observers through the teachers’ lesson plan. The idea of this practice is that teachers build shared knowledge (PO<sub>2</sub>) based on a shared and direct access to the “object” (an instance of DO and MO) to which this knowledge pertains. A similar but more comprehensive form of paradidactic infrastructure can be found in Japanese “lesson study” (cf. Miyakawa & Winsløw, 2009a, 2009b) which comprises also a shared organisation of teacher knowledge and practice in the phase of planning (PO<sub>1</sub>) and, as part of this, a shared elaboration of the lesson plan. Japanese teachers and educators may also say that “open lesson” is a kind of lesson study or a part of lesson study, as the definition of lesson study is not as clear-cut in Japan<sup>3</sup>.

By contrast, this distinction is clearer in the USA: the term *lesson study* is used for a cyclic practice of professional development in which a group of teachers collaboratively plans a lesson, one member conducts the lesson while the others observe it, and the group reflects together on the observed lesson and redesigns the plan (cf. Stigler & Hiebert, 1999). The term *research lesson* is then used for the lesson designed in the process of lesson study and possibly presented to a wider public (cf. Lewis, 2000; Lewis, et al., 2006). A lesson open to the public is called *open lesson* in this paper, regardless of whether it is related to a lesson study. A similar practice is found also in the USA, for instance at the lesson study conferences organised by the Chicago lesson study group<sup>4</sup>. The main difference with the Japanese situation is that in Japan, both lesson study and open lessons are part of teachers’ ordinary experience, rather than activities involving mainly a dedicated minority.

The idea of “open lesson” is certainly not complicated. The fact that it is, in practice, neither obvious or common in most parts of the world seems to be an evident case of what Bourdieu (cited in Chevallard, 2002, p. 11) calls *l’inconscient scolaire* (the unconsciousness of the school): local practices are considered “natural” and alternatives are not even imagined. Our aim here is not to

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<sup>2</sup> Similar formats exist in some East-Asian countries (see e.g., Shen et al., 2007, Quan, 1992 for China), but virtually non-existent in many other countries.

<sup>3</sup> One of definitions given in an encyclopaedia on Japanese education is: “a study whose object is a lesson carried out in school. It includes a clinical study and a study forming its foundation whose objective is to improve the lesson and to develop teacher’s practical abilities” (Hosoya et al. (Eds.), 1990, p. 62).

<sup>4</sup> One can get an impression of the activities of this organisation at the website <http://www.lessonstudygroup.net/>.

describe the phenomenon of Japanese “open lesson“ in general, as most of what could be said in general has been said above and could not reveal the finer details of the format. So instead, we present an in-depth study of one instance of open lesson, based on analyses of videos of the lesson and the following discussion. In particular, we outline the DO planned and realised by the expert teacher, the MO established for children and the PO of the whole session, through which teachers build common knowledge, particularly as regards the techniques of the observed didactic practice.

### *Data and methods*

The *open lesson* to be analysed in this paper was taught on June 25, 2009, in a 2<sup>nd</sup> grade class (pupil age  $\approx$  7 years) of the elementary school attached to Joetsu University of Education, about 200 km northwest of Tokyo. This open lesson formed part of a two day “study meeting” (*kenkyu-kai*), held annually at this school, and attended by hundreds of teachers from all over Japan. During these two days, the school showcases lessons from all school disciplines, as well as other aspects of the school’s life, such as after school musical and sports activities. Each activity contributes to give a holistic impression of the school’s life, governed by the general aim of “preparing students to live in human society” (an approximate translation of the school’s motto). The mathematics lesson considered in this paper was observed by about 70 teachers.

For this study, we collected data from the three phases shown in Fig.1, specifically: the lesson plan (Isono, 2009; its translation is included in Appendix of this paper), the actual lesson, and the post-lesson discussion. The lesson plan was written by the teacher who taught the lesson. A lesson plan, in common Japanese practice, explains the general and specific goals of the lesson, as well as a detailed “script” for the lesson itself, showing its development in time, the actions to be carried out by the teacher, and the expected reactions of students. The lesson plan also situates the lesson with respect to preceding lessons and students’ prerequisites in general. In the open lesson we attended, copies of the lesson plan were distributed to the observing colleagues, so that they could look at it before, during and after the lesson.

The second source of data is the mathematics lesson we observed. The lesson was videotaped and transcribed; also photographs were taken, mainly of students’ productions. The lesson was given in a 2<sup>nd</sup> grade class of 40 students and lasted 65 minutes. It formed part of a series of lessons in this class, which the teacher gave the title “*Sukkiri* by drawing” (see the lesson plan in Appendix). The meaning of this Japanese word *sukkiri* is approximately the *feeling of relief or clarity*, “I got it”. We translate it here as “clarity”. Concretely, after presenting the students with a mathematical problem, the teacher of this class asked the students to “draw the problem” before solving it, in order to experience “*clarity*” in the sense that the problem or their solutions become clearer to them with this drawing. He practiced this unit since the beginning of the school year in April 2009, specifically in every lesson over almost three months up until the *open lesson* we observed. Pictures of some diagrams produced by students in previous lessons were exposed in the back of the class. The third source of data is the post-lesson



discussion organised in the classroom, shortly after the observed lesson; it was also videotaped and transcribed. In Japan, the post-lesson discussion is always organised in the open lesson for the participants who observed the lesson. In this session, there are usually three key persons: the chairman who organises the discussion; the teacher who taught; and a math educator, a university professor or an expert teacher, who collaborated with him or who is just invited to the open lesson. The typical structure of post-lesson discussion is as follows: the teachers' introductory remarks; the participants are invited to provide questions and comments; the discussion develops through a succession of questions, comments and replies, often grouped by the chairman; at the end, the mathematics educator gives a comment on the lesson. The post-lesson discussion we observed in this study was also as we described here, in terms of the participants and its structure.

We conducted the analysis for each source of data described above. In the analysis of the lesson plan (which constitutes the main trace of the pre-didactic organisation (PO<sub>1</sub>)), we identified elements of the mathematical organisation (MO) and the didactic organisation (DO) to be enacted in the actual lesson. Then the actual lesson was analysed in order to show how the MO is established for students in relation to the DO. We identified the mathematical task for students and the techniques they developed, and as far as possible, the theoretical block (technology and theory) of the realised MO. In regard to the DO, we identified the didactic *techniques* of the teacher, and show how they relate to the DO elements of the lesson plan (including its theoretical block). Lastly, we proceeded to the main task of analysing the post-didactic organisation (PO<sub>2</sub>) in the post-lesson discussion and in particular the development of teachers knowledge that took place in this phase, by identifying how the participants' discussion relate to the realised MO and DO, and then by explaining and discussing the corresponding teacher knowledge that emerges from it.

## Findings

In what follows, we briefly present each component of data followed by its analysis.

### *Short presentation of lesson plan*

The lesson plan distributed to the participants takes up two pages and has the title "Clarity by drawing: addition and subtraction". It consists of three sections (see Appendix). The first short section is devoted to an explanation of how the larger series of lessons should contribute to the school's general goal, "preparing students to live in human society", namely: "through interaction with others, pupils build a sense of mathematical values, and based on that they act in society by constructing and modifying their thinking"<sup>5</sup>. In the second section, this general idea is elaborated for the case of word

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<sup>5</sup> "Mathematical values' are the values supporting problem solving in mathematics. Mainly the following three values are considered: 'simplify something which is in a complex state', 'clarify something which is in an ambiguous state', and 'generalise something specific'" (Isono, 2010, p.1).

problems implying addition and subtraction. This section includes the plan of the series, and the students' performance in previous lessons. The third section is devoted to the specific open lesson. Its aim and development in time are precisely described. The development is presented in a table with three columns: expected time duration, main phases and expected student's behaviour, and teacher's support in each phase. Four phases are planned in this lesson: grasp the task; individual drawing and solution; group drawing, solution and presentation; drawing and solution of another task by modifying their drawing with ideas from other groups.

*Analysis of lesson plan (based on methods and theoretical model)*

We identified in the lesson plan several elements of mathematical organisation (MO) and of didactic organisation (DO) planned to be enacted in the actual lesson. In what follows, we first present them, and then discuss the pre-didactic organisation (PO<sub>1</sub>) evidenced by the lesson plan.

The planned MO appears especially in the table of development of the lesson. The main mathematical task  $t$  planned to be proposed in the lesson is: *There were 16 passengers in a Daido bus. Later, some passengers got on this bus. Now as a whole, the number of passengers is 34. How many people got on later on?* This is a problem located in the section "Mutual relationship between addition and subtraction" of the area "Mathematical relations" in 2<sup>nd</sup> grade of the Japanese national curriculum (MEXT, 2008, p. 83). At this point, we can identify the mathematical praxeology to be developed through the task type  $T$ : *Given whole and a part, find the other part*. Considering this a problem of arithmetic, we could represent this as  $w=p+x$  where  $w$  and  $p$  are given whole numbers and  $x$  is to be found. This might lead to techniques such as the rewriting  $x=w-p$  with a technology on 'moving elements of an equation to the other side', 'subtracting the same thing', etc. However, at this level and depending on the context, other ways of representing the task and its solution might appear, such as representing the whole as  $w$  dots, circling  $p$  of them, and counting the uncircled ones to get  $x$ . This is still an abstraction from the concrete task that would not be immediate to students. The technique expected by the teacher is making a drawing representing the problem situation, and then based on this, writing a formula<sup>6</sup> of subtraction. The teacher also anticipates another technique, which leads to a wrong solution: writing a formula of addition instead of subtraction due to the expression "as a whole" in the task statement.

The table of the development of lesson also allows us to identify several planned didactic tasks and techniques (elements of the practical block of the DO). For example, in the first phase of grasping the

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Notice also that Nunokawa (2005, p.333), who was collaborating with this teacher, relates the Japanese notion of *mathematical values* to the Anglophone notion of *socio-mathematical norm*.

<sup>6</sup> The Japanese term *shiki* is translated here into 'formula'. But in fact *shiki* is an expression containing only number symbols, operation symbols, eventually empty boxes or letters: e.g., '34 - 16 =', '16 + □ = 34'. Quite often, *shiki*, not only the solution, is required to be written by students, as they solve word problems in elementary school.

mathematical task  $t$ , the teacher plans not to give the task statement as a whole, but to make students find the last sentence: *How many people got on later on?* This is a didactic technique to make students grasp the mathematical task. After the second phase of individual drawing and solution, group work is proposed: each group must produce a common diagram on a sheet of magnetic white board. The procedure of having them draw, and then discuss and draw a common diagram, is a *didactic technique* for the *didactic task* of helping students construct a mathematical technology related to the mathematical task type  $T$  and techniques to solve it, while reflecting critically on the concrete problem rather than mimicking some standardised technique for  $T$ . Regarding the management of the whole class presentation, the teacher formulates some techniques explicitly in the column of *teacher's support* of the table. Three of them are:

$\tau_1$  : Ask the students about “what becomes clear through their drawing” (to help students be explicit about mathematical value of representations);

$\tau_2$  : Write on the board the idea of the group that led to a drawing which achieves “*clarity*” (to help students express the essential of their shared knowledge);

$\tau_3$  : Group the magnetic sheets [with students' collective drawings] according to their idea (to help students distinguish different ideas).

All these techniques serve to amplify and explore students' ideas, as they present their drawings of the mathematical task  $t$  and its solution. This way the drawings help to enrich the students' mathematical technology related to  $t$ , and in particular the degree to which it is shared in the community of the class. The corresponding didactic goal or task is to help students “conceive the structure of addition and subtraction by drawing the relation between numbers given in word problem, and use it as a rationale to establish a formula” – this sentence is given in the beginning of the third section as an overall aim of the lesson.

Finally, in the first and second sections of the plan, we find some elements that belong to the theoretical block of the didactic organisation (DO). The general idea of the lesson mentioned above (“through interaction with others, pupils build a sense of mathematical values, and based on that they act in society by constructing and modifying their thinking”) belongs to the theoretical block of the DO to be realised in the actual lesson. For instance, the didactic technique of group work which is planned in a third phase is directly justified by the sentence “through interaction with others”. And the fourth phase of lesson, accomplishing another task by modifying their drawing and using ideas from other groups, should achieve the main goal of the sequence, to help students “modify their thinking”. One could also notice that the general theory of the lesson implies general didactic tasks, such as helping students “build a sense of mathematical values through interaction with others”. Similarly, the second section of the lesson plan mentions three reasons for the lesson series (not a particular lesson). They correspond roughly to its development in four phases: to help students

“understand” a problem situation even if they are “weak in word problems”; to help students find out the “relation between numbers” in the problem; and to help students “reflect together” using the drawings as representations of the problem and ways to think about it. Here, the teacher also emphasises the importance of the students moving from naïve depictions of the situation (such as a bus with passengers) to “drawings showing the relation between numbers”. This organisation of the teaching project in “global tasks” clearly belongs to the theoretical block of the didactic organisation for the series of lesson, as it justifies and explains the more detailed planning of the didactic practice to be realised.

Thus, the lesson plan not only serves to show the teacher’s practice foreseen in the classroom (i.e., didactic tasks and techniques), but also outlines the theoretical block of the teacher’s practice, by *explaining the main didactic techniques* and by *justifying them with respect to more or less general objectives*. In terms of the pre-didactic organisation (PO<sub>1</sub>), teacher’s pre-didactic task is therefore not just to plan a lesson, but also to establish the theoretical block of the DO. It should be noted that the teacher collaborated, in this phase, with his colleagues and a mathematics educator from the university. As we will see later, a main point of the post-lesson discussion is to develop this theoretical block further *in the community of participants*.

#### *Short presentation of lesson*

The lesson we observed almost followed the table of development of lesson given in the lesson plan. Only time did not allow for the fourth phase the teacher had foreseen in the lesson plan. After the first seven minutes of informal discussion, the mathematical task is introduced as it was in the lesson plan. After about five minutes for individual drawing, the 40 students are organised in 10 groups. Each group produces a common diagram on a sheet of magnetic white board (each group gets one sheet). The groups work for about 15 minutes. The diagrams are then exposed at the blackboard one by one in the order designated by the teacher, who calls on the “chair” of each group to explain it to the class, while other students frequently join in with comments. The teacher conducts the whole class discussion on the 10 drawings for the last 25 minutes of the class. At the end, the teacher concludes the class by explaining the “trap” of the problem:

Teacher: If you have “as a whole”, it is usually an addition, however, after you all were trying and saying many things, you got 18 people. Saying it is 18, you were trying to represent the use of subtraction in the drawing, and you found it easier.

#### *Analysis of lesson (based on methods and theoretical model)*

We now try to describe the didactic practice and mathematical practice which unfolds in the lesson. Theoretical blocks of these practices will be discussed as far as our data permit. In the first phase, one could identify the didactic technique and mathematical task that are foreseen in the lesson plan. The teacher dictates the task without the last sentence, while writing on the blackboard: *There were 16*

*passengers in a Daido bus. Later, some passengers got on this bus. Now as a whole, the number of passengers is 34.* When the students have copied this text into their notebooks, the teacher proposes, orally, the following task ( $t_1$ ) to the students: What would be the next phrase? After about 2 minutes of whole class discussion, the class agrees to consider the main mathematical task ( $t$ ), also dictated as before: *How many people got on later on?* For the individual work (second phase), instead of asking them to answer the question, the teacher asks the students – as in other lessons of this series – to first make a “drawing” (Japanese: *e*, a free drawing) or a “diagram” (Japanese: *zu*, clearly distinct from a free drawing, yet not indicating a precise form) that gives them “clarity”. This is a didactic technique which was not explicitly mentioned in the lesson plan, but was easily implied from the reasons (theoretical block of DO) for the series of lessons “Clarity by drawing”.

In the group work (third phase), students are specifically instructed “do not write a formula” (calculation), but to “make a drawing that gives you clarity” (about the problem). It is clear that the main mathematical task is still  $t$ . The procedure of having them draw and then discuss is, as already mentioned, a didactic technique for the task of helping students construct a mathematical technology related to T, as well as techniques to solve it.

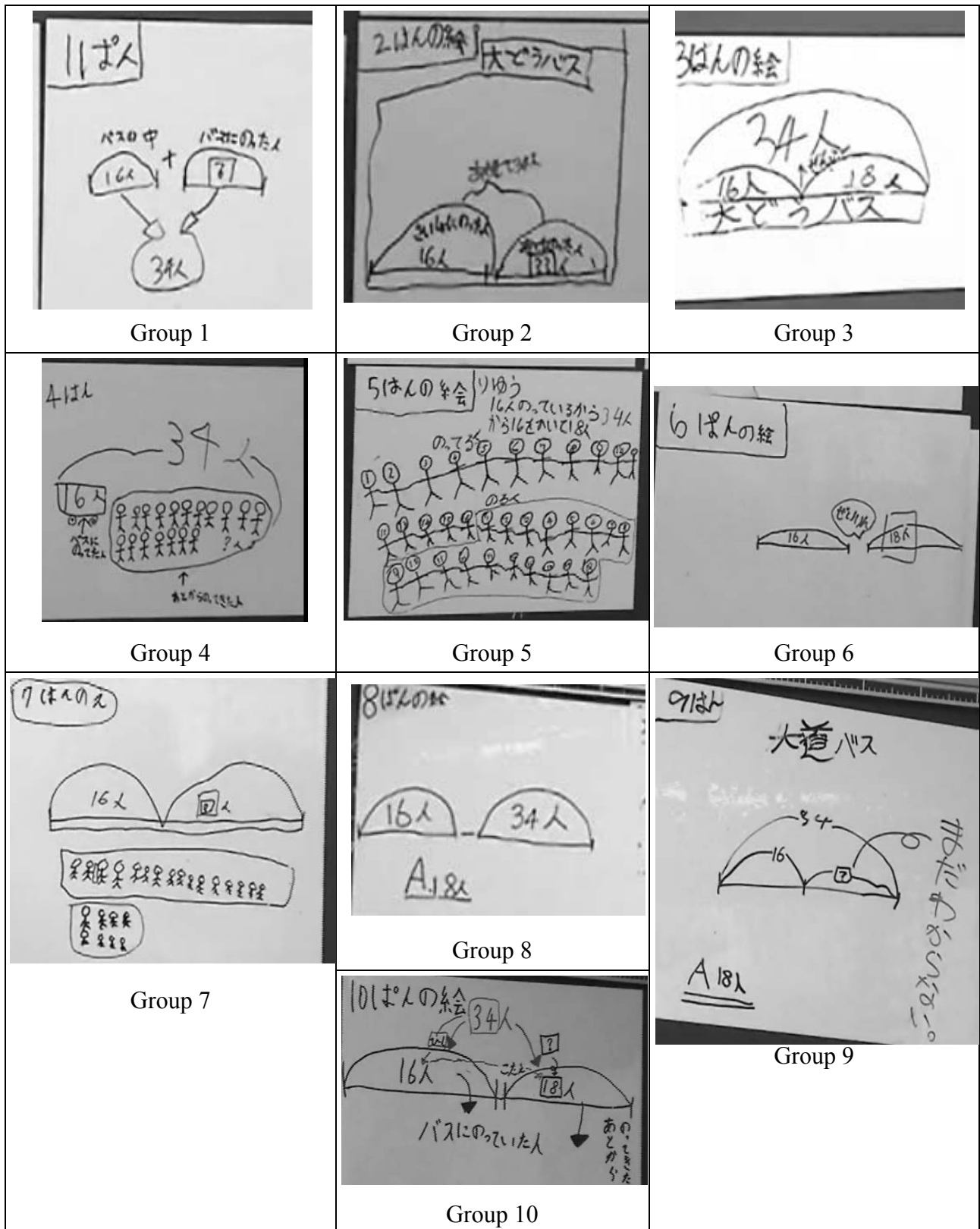
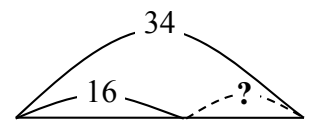


Fig. 2: The diagrams produced by the 10 groups of students. NB: the text to the right on the drawing of Group 9, which means “we don’t know yet”, was added by the teacher during the whole class discussion at the end of the lesson.

The students produce a relatively large variety of diagrams (see Fig. 2). Some groups (for instance group 5) produced a more “naturalistic” representation of the situation, with pictogram like “people” in two groups making up 34, and in which you can count; for example, “how many must be added from the 16 to get 34”. All other groups have produced drawings which bear some resemblance to what Japanese mathematics teachers call “segment diagrams” (*senbun zu*). A *segment diagram* is “a diagram to facilitate a visual grasp of numerical quantities, some of which may be unknown, and also their mutual relations according to a problem statement, by means of their representation as a length of a segment” (JSME, 2009, p. 213). For instance, the segment diagram shown in Fig. 3 represents the addition of 16 and an unknown number, with the result being 34. Representing addition in this way is an object of teaching in Japanese elementary school mathematics, and expected to be learnt as a technique to translate the problem situation to a formula (MEXT, 2008, p. 83). It may also be used as a didactic technique to prepare the teaching of equations in lower secondary school. We notice in passing that the *naming* of this technique is part of the shared didactic technology of all Japanese mathematics teachers at this level and thus a part of the paradidactic infrastructure (cf. the next section).



**Fig. 3: segment diagram.**

Besides these drawn parts, several of the diagrams contain text, such as the upper part of group 1, which reads: (to the left) “people in the bus” and (to the right) “people who got on”. Finally, two drawings contain an arithmetic operation (+ or –), which in a sense violates the teachers instruction not to write calculations. The drawing of group 8 also shows that the arithmetic symbolism related to “minus” is not fully mastered by the students: they realise that a subtraction should be done, but put 16 first and then 34, corresponding to the order in which these appear in the problem statement. The oral explanations of the students develop a *technology* from their drawings, for example, (referring to the diagram of group 5): “*The people rounded by a red rectangle are the ones who are going to get on, um, then, the people who are not rounded by a red rectangle are the ones who are in the bus*”.

In the whole class discussion, the teacher first focuses on the presence, or absence, of a question mark or other indication showing the unknown (the number of people getting on the bus), and also on whether the drawing gives a clue about how to find it. The groups who seem to favour addition (e.g., group 1) are taken forward first, then the drawing that, on the other hand, clearly shows the problem (Group 9). After this, the similar drawings are briefly discussed together: the ones which offer various ways to “count up” to 34, in order to get the result. The teacher goes back to the drawing of group 8 and says: “Group 8 wanted to subtract. They probably knew in their mind that it is a subtraction...”. He finally takes out group 6’s drawing and marks the number 18 (meaning 18 people), saying that “they were discussing that this is a question mark”, and points out that they wrote, between 16 and 18, “whole” (i.e., the total number of passengers).

The teacher’s didactic techniques in conducting the discussion can be summarised as follows:

focusing on the different potentials of the drawings to represent the task  $t$  in a way which helps “seeing” efficient techniques. Since the drawings are not all self-explanatory, the students’ explanations and comments – that is, their emerging mathematical technology – become key drivers of the debate. As one will often find in this kind of teacher led debate, the teacher refrains from explicitly validating students’ production. It could be added that the students seem to agree to the answer while the technique for obtaining it could at least be obtained through different modes of thinking (including the more naïve representation, whose technique amounts to “counting up”: e.g., group 5). In the next section we will explore more deeply its rationales and the ways the observation of the class led teachers to discuss them in a wider (and emerging) theoretical setting.

#### *Short presentation of post-lesson discussion*

The post-lesson discussion is held just after the observed lesson and after the students left. Some of the participants are seated at the children’s desks while others sit or stand around them (Fig. 4). At the teacher’s desk, we find the teacher, a chairman (teacher from another school in Joetsu) and the university professor who collaborated with the teacher. The post-lesson discussion we observed lasted about 50 minutes. It was structure as follows. After the chairman’s welcome, the teacher gives some introductory remarks on the lesson and its context. Then the main part of the agenda comes: the participants are invited to provide questions and comments. The teacher reacts to most of them. The discussion involves 10 of the 70 attending colleagues, some of whom speak more than once. It develops through a succession of questions, comments and replies, loosely grouped by the chairman as he occasionally asks for “related questions” before letting the teacher reply. The discussion with participants lasts about 40 minutes. At the end, the university professor gives some concluding comments.



**Fig. 4. Scenes from the post-lesson discussion**

#### *Analysis of post-lesson discussion (based on methods and theoretical model)*

The post-lesson discussion is a paradidactic organisation for the community of teachers. In what follows, we first identify how the mathematical organisations (MO) and didactic organisations (DO)



are referred to in the teacher's introductory remark and in the discussion, and then show how the format of post-lesson discussion provides to the community of teachers a frame for post-didactic practice where they develop teacher knowledge in relation to the observed MOs and DOs.

*Post-didactic practice: teacher's introductory remarks*

The teacher first explained the fundamental motivation (a didactic problem) which made him develop the lesson series "Clarity by drawing":

In upper grades, the text book uses a lot of segment diagrams and other diagrams, to explain something. But I asked myself whether children really understand the diagram and use it as a tool for understanding.

He then explained the idea behind the lesson sequence, including the observed lesson:

In the progression of the course, at the beginning, children get "clarity" when they draw a thing. They get "clarity" when they depict a representation of the scene<sup>7</sup>. (...) Some time later, since I have the goal to take them to band diagram or segment diagram, I gave some problems they cannot solve well by just making a drawing. (...) Today we have worked with the unit<sup>8</sup> of *inverse calculation with holes* that we usually cover at the very end of the third trimester of 2<sup>nd</sup> grade. So far, children were working on a drawing of addition or subtraction, and if they put things together, it was addition. (...) Today, one can read the problem statement as indicating addition, but once you actually make a drawing, you see that there is a subtraction.

The teacher indicated here a didactic technique  $\tau_4$ , related to the task of having "children really understand the diagram and used it as a tool for understanding":

$\tau_4$ : to adapt the choice or form of the mathematical task to children's capabilities of illustrating the task by a drawing.

The first tasks that were given could be illustrated by simple, naturalistic depictions of the situation in the word problem; then the students progressively got tasks which are harder to illustrate. The teacher's explanation thus clarified not only the practical block (tasks and techniques) of previous didactic practices (including today's lesson), but also the theoretical block (technology and theory) of these practices.

*Post-didactic practice: discussion with participants*

The discussion with participants is the main post-didactic practice. To show the flow and contents of the discussion with participants, we provide a chronological paraphrase and analysis of the contributions of the participants (numbered P1, P2, ...) and the replies of the teacher (T).

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<sup>7</sup> Here, the 'scene' refers to the context of a word problem.

<sup>8</sup> The "unit" refers to the textbook used in this class.

P1 asked two questions in the beginning of the session. The first one is: “what does it mean precisely that the students experience “clarity”?” T provided the following rather precise definition to this question:

“Clarity” is to have a drawing that helps them to get an answer or to make a formula, or helps them to see what to do using a question mark, instead of a drawing that just describes precisely the problem situation.

This definition shows the teacher’s expectations on how students use diagram (representation) to solve a word problem: the diagram is used as a tool to establish a formula (mathematical technique) or to justify the formula (mathematical technology). The teacher also emphasised that (in connection to the realisation of  $\tau_4$ ) there has been a *progression* through the series of lessons from just drawing the situation naturalistically, to actually using the drawing to get a clear and useful representation of the problem. These ideas which belong to a theoretical block of didactic practices are already mentioned in the lesson plan, but detailed here in relation to concrete examples of diagrams given in the observed lesson and to previous mathematical practices in the series of lessons.

P1’s second question was about segment diagrams: “Eight of ten groups use segment diagrams. What did you teach about this?” We recall here that segment diagrams is a specific form of representation which is used mainly in higher grades, and so its appearance in the students’ drawings (Fig. 2) motivated P1 to ask about T’s previous didactic practices and its relationship with the mathematical organisation identified in the observed lesson. T replied:

Actually, I did not teach about segment diagrams so far, students use ideas they saw or heard elsewhere<sup>9</sup> (...) they just start using drawing so as to get “clarity”. (...) some students still consider that the diagram shows a “mountain” [pointing the drawing of group 6], (...) they do not think completely that the length of segment is at stake.

T did not give a unique reason for the students’ use of segment diagrams, but instead he explained how the actual state of students’ use of diagram had evolved gradually through previous lessons. The explanation focuses on students’ mathematical practice rather than his teaching practice which, apparently, has not been strongly directive as regards the use of segment diagrams. However, we see already from the introductory remarks of T, cited above, as well as from comments cited below, that one of his aims is indeed to prepare the students for the use of segment diagrams in later grades.

P2 asked another question: “Can you relate today’s lesson more precisely to the school’s motto?” Then as a related question, P3 mentioned a number of moments in the lesson where students seemed to learn, and asked if the teacher noticed other particular moments where the students really gave evidence of progress. These are questions and comments about the relationship between the

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<sup>9</sup> At this age, it is uncommon to attend a private mathematics class (*juku*) outside of school time, but it occurs.

theoretical block of the didactic organisation, and the didactic and mathematical practices observed in the lesson. As a response, T mentioned instances where children validated each others' methods, and claimed that it is important that students create the drawings together in groups, and afterwards discuss the quality of the drawings:

I consider that the situation where this kind of thing is created, modified and enhanced due to children's exchange in the classroom helps preparing them to live in human society...

The teacher's answer concretely connects the mathematical practice observed in the lesson, in particular the process of elaborating the mathematical technology, to the school's motto (an element of didactic theory – supposedly present throughout the institution). It shows how a theoretical idea can be implemented in the actual lesson.

Next, P4 and P5 challenged a specific didactic technique observed in the lesson. P4's comment could be summarised as: while T insisted a lot on drawing before writing a formula to solve the problem, to get "clarity" this way, some students seemed to use the answer to make their drawing (for example group 4). As a related comment, P5 mentioned that the national curriculum for mathematics stresses the use of different representations (words, diagrams, formulae...) and that some children may get "clarity" through the use of words or formulae. P5 suggested that one could reconsider the choice to ask the students to follow a particular order, like first drawing and then calculating, etc. The didactic technique questioned here is the teacher's instruction not to write a formula, which insists that students follow a particular order of mathematical techniques. The basis for this question is the participants' observation of mathematical practices in which some students did not necessarily get "clarity", and also another element of the theoretical block of the didactic organisation (national curriculum). T did not explicitly address the suggestion of P5 that for some children, beginning with a drawing might be less natural. Instead, he explained the rationale of his technique of focusing on the process of understanding the problem and having it reflected in the students' drawings, so that subtraction appears naturally:

T: Because there are still some drawings that don't become a diagram of subtraction for children, we are doing a final verification with this drawing [the diagram of group 5]. And, I wanted to lead children to realise that one can get the same meaning with this kind of drawing [diagram of group 1] or with this kind [diagram of group 5]. I should have used a question mark here [diagram of group 5] too, though. I wanted to lead them to realise that with this drawing [diagram of group 5] one can say it [there is a subtraction] in a real scene<sup>10</sup> as well.

After T's reply to P4 and P5's comments, several participants (P6, P7, P8, P9, P2) consecutively reported observations on students' difficulties, without T's intervention. P6 pointed out some students' (erroneous) mathematical technique leading to write  $16+34$ , which appeared during their individual

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<sup>10</sup> See footnote 7.

work, and that the group work eliminated. Then he suggested an alternative didactic technique to bring both techniques (34–16 and 34+16) forward to the whole class, in order to create a stronger necessity of making drawings, to decide which technique is correct. P7 and P8 agreed with P6, and provided additional comments. P7 pointed out the spontaneous technology behind students' mathematical technique, the use of addition, in the first individual stage: they feel it's about addition, because in a sense it is (there is a total of two parts). P8 mentioned the role of drawing with respect to different mathematical practices: drawing both to find and show a solution as well as to demonstrate an answer to be false. These comments explore and discuss the mathematical organisation at stake in the lesson. Then, P9 suggested sharpening the didactic technique  $\tau_2$  to help students draw the problem as it is formulated on the blackboard: as the drawings are presented and discussed, it could be made more explicit *what parts of the problem formulation* correspond to *what elements of the drawing*. On the other hand, P2 disagreed with P6-P8 and said that because of the whole series of lessons, children are already in the habit of using drawing as a first step of their work, so they don't need to be told. He also asked if this method would be used in other parts of the curriculum. In his response, T underlined that it is a point to help students use a diagram as a tool to solve problems, and not just as an illustration of the solution. He did not know yet about the extent to which this method will be pursued in other parts of the curriculum, but he hoped it will. As a last contribution from the participants, P10 asked a question about the didactic infrastructure, specifically the impact on didactic practice of the length of the lesson (longer than in her school). T replied that the children are accustomed to the long lessons and that he focuses especially on taking advantage of the long period to allow enough time for group discussions.

At the end, the university professor developed two points: the didactic process he observed (in previous lessons) regarding the students' diagrams, going from cartoon-like "drawings" to more precise diagrams showing relations and also the difference between what is known, and what is to be found (again, corresponding to  $\tau_2$ ). Moreover, the methods of drawing before calculating may promote a more reflective and shared approach (theoretical block of didactic organisation):

(...) when a problem is given, there would be children who quickly solve it and get "clarity". Like [one of the participants] told us earlier, children may assume "clarity", even with a wrong answer. When this happens, sharing it with other people and asking themselves what is really a problem, and discussing with them, and thinking what to do (...) is one good experience they may have. In that sense, I expect that the children grow up who can stop to think, and the Prime Minister will be also one of them.

#### *Post-didactic practice: development of didactic theoretical block*

While the discussion as a whole is obviously quite loosely structured, it is far from unfocused. It is almost entirely about theoretical justifications – or critiques – of the didactic practice of the lesson observed just before the discussion. The didactic techniques are justified or questioned with reference to:

- overall goals of the school's teaching, primarily its motto (which is clearly a central element of theory for didactic practices in this institution, almost like an axiom);
- long term goals related to the curriculum, where abstract diagrams are increasingly used to illustrate quantitative relationships and equations, but perhaps without much 'understanding' on the part of the students;
- the concrete didactic task of developing students' sense of the difference of addition and subtraction;
- students' autonomous use of non-discursive representations, which is expected to emerge, like segment diagrams, as shared mathematical techniques related to tasks on numerical relations and operations.

In particular, didactic techniques to *manage the sharing and validation of children's methods* are important as their drawings become central to the mathematical technology produced and shared by students. It seems to be the point of several comments that the delicate transition between grasping the mathematical task, choosing or devising a technique, and justifying it to others, could have been given different organisation and many comments relate to the role of the drawing activity.

There is no doubt that both the succinct lesson plan and the shared observation experience are crucial to enable the discussion to explore a "real" didactic theoretical block. The term "clarity", central to the design of the lesson series, emerges as a central element of the technology in the discussion, and the participants seem to accept the teacher's definition (cited above). This common technology permits a strong focus on:

- the didactic techniques for achieving "clarity" (observed in the lesson, and also alternative techniques);
- their justification according to the theoretical levels.

Our analysis illustrates the essential connection between pre-didactic organisation traced in the lesson plan, the observed didactic practice, and the post-didactic practice of the discussion: the latter is to explore the theoretical block of the didactic practice, already indicated in the lesson plan. In general, the main task of the discussion in an open lesson session is *to develop and explore the theoretical block of the didactic practice observed in the lesson* (already initiated by the teacher in the lesson plan). Among the techniques we can observe in the post-didactic practice, we emphasise the organisation and tone of the discussion. The discussion is organised by the chairman with explicit and different roles assigned to the teacher, the university professor and the participants. The discussion strictly focuses on the didactic techniques of the lesson, and their relations to explicit theoretical principles. It is also important to notice the respect shown to the teacher during the discussion. Participants who ask questions and comments stand up and begin with presenting

themselves and thanking the teacher for being allowed to attend his lesson (cf. pictures given in Fig. 4). All comments are profoundly courteous. And after the comment from the university professor who worked with the teacher, the chairman closes the discussion as follows:

(...) we had a lot of valuable comments from you all. We are going to incorporate them into the future work. Thank you. Well, I'm going to close this meeting now. I would like to close it by giving applause to Mr. Isono who provided this open activity today. [applause from participants] So, let me close this meeting now. Thank you to everyone.

The tone of the discussion cannot fully be reflected in English translation as certain forms of courtesy in the Japanese language are hard to render. But we think that it is an important condition for the qualities of the post-didactic organisation that the teacher, who opens his lesson as an opportunity for learning, is treated as an expert of the lesson and by no means as someone to be “attacked” or criticised.

## **Discussion and conclusions**

The “open lesson” practice is a form of teacher knowledge development which is well established in Japan. In this paper we have described and analysed its main elements:

- the lesson plan of the teacher (pre-didactic practice) which the participants of the open lesson may study before the lesson;
- the real-time observation of the lesson;
- the discussion after the lesson (post-didactic practice).

An open lesson session has been described as a being primarily a specific form of *post-didactic practice* related directly to a lesson and aiming specifically at elaborating the theoretical block of this didactic practice. It is also a main point of this paper that while teachers' practice can always be said to consist in the three phases shown in Fig. 1, they often remain private (enacted by a single teacher). As a result, one may not find much basis for sharing didactic theory related to concrete didactic practice.

The practices of open lessons are not strongly codified, explained or justified in the Japanese school institution, where they appear more as a naturalised tradition for the teachers' professional development and for the improvement of teaching. There is no explicit theoretical block of the post-didactic practices related to open lessons. The existence of these practices does not require official explanations and justifications in terms of their contribution to the professional development for the teachers who participate in them: it just appears as a “naturalised” practice<sup>11</sup>. Admittedly, we cannot

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<sup>11</sup> In the professional literature for Japanese teachers, one can find some descriptions of the practice and theory of lesson study (e.g., Inagaki & Sato, 1996). As an “open lesson” can be seen as similar to parts of lesson study, one may say that it is not completely without an explicit theoretical block.

say much about the general effects and impacts of the paradidactic infrastructure of “open lesson” based on the single case which was presented and analysed in this paper. But the commonness of open lessons in Japan, together with the absence of exterior or material motivation for this practice, strongly indicates that Japanese teachers find this practice useful and professionally rewarding, and it is the nature of this experience that we exposed and analysed here.

In order to conduct our analysis, we have developed a new technology *about* the open lesson practice and some elements of theory as well. In particular, our study suggests how open lessons, as a shared *paradidactic* practice, can contribute to elaborate common theoretical blocks related to *didactic* practices. In the context of the “open lesson” studied here, we have merely indicated how a germ of didactic technology, the discourse related to “clarity” and the means to achieve it for young students, may develop and be shared in a wider community of teachers. It is unquestionable that Japanese mathematics teachers have developed a remarkable level of shared didactic theory, as illustrated by the existence of entire dictionaries of mathematics teachers’ terms (didactic technology), such as JSME (2009). Our case suggests a hypothetical and partial answer to the intriguing question of *how* these shared didactic theoretical blocks develop.

With just a general description of what open lessons are, one might get the impression of an activity which is narrowly focused on a particular lesson, entirely conditioned by specific topics, students, and other conditions. Our case study shows that much of the discussion and potential learning for teachers is of a more general nature. While the concrete lesson and its didactic techniques (such as  $\tau_1$  -  $\tau_4$ ) could certainly be of intrinsic interest to them, we have shown that the discussion relates the lesson to more theoretical aspects of the mathematics curriculum as such, and even to more general pedagogical and societal aims of the school. This way, the discussion provides a space – an “ecology” in the sense of Chevallard (1988, p.99) – for developing teacher knowledge that is *neither* narrowly limited to teaching a particular lesson *nor* drifting into discussions of teaching philosophies which are more or less detached from the reality of schools and teaching. And, just as in the case of lesson study, the goal is not primarily aimed at revising or refining the didactic practice in the observed lesson, but to develop and strengthen the shared theoretical blocks relating and informing didactic practice in a much wider sense.

In conclusion, the present study indicates *why* open lessons represent, to Japanese teachers, an attractive element of a professional learning community of teachers (cf. Richmond and Manokore, 2011), and that the communal learning arising from this practice goes largely beyond what can be said about a single lesson. We hypothesize that the establishment of similar paradidactic infrastructures could be an important contribution to enhance the professional development and status of teachers in many Western countries. Didactic practice – in Japan as in other countries – most often

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But the practices of lesson study or open lesson developed in Japan without such a literature and could continue to exist without it.

involves just one teacher, a state of affairs which seems to prevent the development of shared theoretical blocks for that practice. Semi-private, “bricolage”-like practices rarely achieve high professional status and the knowledge of the practitioner disappears with him. And that is ultimately why the study and development of paradidactic infrastructures like “open lessons” could be of significant importance to the teaching profession.

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## Appendix (lesson plan translated into English)

### 2<sup>nd</sup> grade Mathematics Activity Plan

#### *Sukkiri* [Clarity] by Drawing! : Addition and Subtraction

Teacher: Masato Isono

#### 1. “Preparing pupils to live in human society” in this activity

Through interaction with others, pupils build a sense of mathematical value<sup>24</sup>, and based on that they act in society by constructing and modifying their thinking.

#### 2. About the activity

##### (1) Goal

By drawing the problem situation which calls for addition or subtraction, understand the situation and the relation between numbers, and use it for solving the problem.

##### (2) Reasons for this activity

The activity is to solve word problems by drawing the relation between numbers included in the problem statement, and to improve the way of drawing.

There are three reasons for this activity.

The first reason is that drawing the situation given in the word problem helps pupils to understand the problem situation. Pupils who are weak in word problems cannot often understand the problem situation after reading the problem statement once. Drawing at the same time as reading the statement allows them to organise numerical values, to reconstruct the problem situation by means of their own drawings or words, and therefore to understand it.

The second reason is that drawing the situation leads pupils to find out the relation between the numbers involved, and their structure. For example, in the situation of “There are two apples on each plate. There are three plates. How many apples are there as a whole?”, pupils will be able to conceive the structure of multiplication – that there are three pairs of two apples – after drawing the plates.

The drawing of numbers which appear in the problem also leads pupils to realise the advantage of taking 10 objects as a group. For example, when drawing 14 candies, drawing a group of 10 candies and then 4 candies will make it easier to count them. This idea of taking 10 objects as a group will be used later in the written calculation of addition and subtraction.

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<sup>24</sup> See note 5 for the exact meaning of this term.

The third reason is that whole class reflection on a drawing leads pupils to formulate their sense of mathematical value and to see the importance of it in the classroom. The reflections could involve, for example, that if two objects to be added have different size, it can be advantageous to show the difference in the drawing; if the objects are scattered in the drawing, it may clarify the situation to pull them together; if there are many objects, it helps to represent them by grouping some of them.

The pupils' sense of mathematical value will be built up in a process of producing a clear representation of a mathematical phenomenon in the problem solving situation, and in a process of realising the advantage of representing something in a general way. Through such experiences, one can create an ambience in the classroom which allows pupils to reflect using their sense of mathematical value.

### (3) Activity plan and summary of previous activities

<First trimester> 34 M<sup>25</sup> as a whole

Phase 1 (10 M)

Realise that a drawing showing the situation of a word problem makes the problem easier to understand.

Phase 2 (10 M)

Realise the difference between a drawing showing the situation of a word problem exactly, and a drawing showing the relation between numbers.

Phase 3 (6 M)

In the word problems of addition and subtraction, be able to make a drawing showing the relation between numbers.

Phase 4 (8 M) Today's lesson: 27<sup>th</sup> and 28<sup>th</sup> M

In the word problem using a box ( $\square$ )<sup>26</sup>, be able to solve the problem by means of a drawing showing the relation between numbers.

In Phase 1, pupils learned that looking at a drawing showing the situation of a word problem makes it easier for them to see the situation and to get a solution than if they just read the problem formulation,.

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<sup>25</sup> "M" means *module* which is a time unit of 30 minutes. In this school, two modules are used for each mathematics lesson.

<sup>26</sup> Empty box is used to show an unknown number in Japanese elementary school.

In Phase 2, the activity involved mainly word problems for which a formula is not easily established, such as *wasazan*<sup>27</sup> and problems related to order<sup>28</sup>. To obtain the right answers in these problems, it is not enough to just add or subtract one by one the numbers appearing in the problem statement. Through the experience of solving this kind of word problems, pupils knew that there are some cases where it is not helpful to establish a formula as the first step of solving a problem. And, in the second half of Phase 2, more abstraction in the drawings was observed as pupils started drawing something similar to a segment diagram<sup>29</sup>.

In Phase 3, pupils started realising that the drawing which is most helpful for solving a word problems is not always one on which the situation is precisely drawn, but often it is one showing the relation between numbers. In addition, they realised that a drawing with a box (□) or a question mark (?) tells them what to do and is therefore helpful for problem solving.

### **3. About this lesson**

#### **(1) Goal**

Be able to find the structure of addition and subtraction by drawing the relation between numbers given in word problem, and use it as a rationale for establishing a formula.

#### **(2) Principle ideas for the development of lesson**

In this lesson, pupils try to solve a word problem in which the expression “as a whole” is used, while the operation to be done is subtraction. Therefore, the lesson will be developed by focusing on the fact that there are some cases where “as a whole” does not imply the use of addition.

In the first half of the lesson, individual work comes first, and then group work. First of all, I will let pupils make a drawing that gives *clarity* individually and allows them to establish a formula. Then I set up an occasion for each pupil to modify their thinking by means of a group discussion with their own individual drawings. After this, in order to communicate the idea of the group to others in the classroom, pupils prepare a common drawing which is more appropriate for the communication, and then present it to the class.

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<sup>27</sup> English translation could be “Sum and difference problem”. When the sum and difference of the cardinalities of two sets are known, find the cardinality of each set. For example, “in a class, there are 41 students, and there are 9 more boys than girls. Find the number of girls and the number of boys. In junior high school, this is usually solved by the system of equations while in primary school, more informal techniques are used.

<sup>28</sup> The problem with ordinal numbers for which just adding the given numbers leads to an incorrect answer: for example “some children are standing in a line. Taro is the 5th from the front and the 7th from the back. How many children are there as a whole?”

<sup>29</sup> This term is explained in the “Findings” section.

In the second half of the lesson, pupils make a drawing and solve a new word problem by borrowing the ideas presented by other groups or by clarifying the idea of their own group.

**(3) Development of lesson: 27<sup>th</sup> and 28<sup>th</sup> of 34 M (65 min)**

<b>Time</b>	<b>No.: pupils' activity; ●: expected behavior of pupil</b>	<b>○ Teacher's support</b>
5	<p><b>1. Grasp the task</b>  <b>Task</b> There were 16 people in a bus. Later, some people got on this bus. Now as a whole, the number of people is 34. <u>How many people got on later on?</u></p> <ul style="list-style-type: none"> <li>● Grasp the task by imagining the problem situation</li> <li>● Try to make a drawing in order to understand the problem situation</li> </ul>	<p>○ When introducing this problem, do not show the underlined sentence and make pupils guess what kind of sentence will be there.</p>
15	<p><b>2. Make a drawing individually and accomplish the task</b></p> <ul style="list-style-type: none"> <li>● Based on the expression “as a whole”, find an answer by addition.</li> <li>● Establish a formula using subtraction.</li> <li>● Make a drawing of the whole and the part.</li> </ul>	<p>○ For pupils who get stuck with drawing the situation, support them to make a concrete drawing.</p>
25	<p><b>3. Make a drawing in a group, accomplish the task, and present it</b></p> <ul style="list-style-type: none"> <li>● By drawing a bus and the people, make the problem situation easy to understand.</li> <li>● Abstract the number of people and represent it by “○” or a band<sup>30</sup>.</li> <li>● Identify the situation of subtraction on the basis of the relation between part and whole.</li> <li>● By using a question mark or an empty box, identify that the situation requires to find a part, so it is about subtraction, and solve the problem.</li> <li>● Realise that there is a case where subtraction is used even when the expression “as a whole” (implying addition) appears.</li> </ul>	<p>○ Distribute the folder including drawings made in previous activities.</p> <p>○ Make a common drawing of the group on a magnetic sheet.</p> <p>○ At the time of presentation, make pupils talk about what becomes clear by making a drawing.</p> <p>○ At the time of presentation, group the magnetic sheets which are based on the same idea.</p> <p>○ Write on the board the idea of the group (who made the drawing) that makes <i>clarity</i>.</p>
20	<p><b>4. Accomplish a task by modifying the drawing of their own group or by borrowing the ideas of other groups</b>  <b>Task</b> There were some people in a bus. Because 18 people got on this bus later, the number of people is 37 as a whole. How many people were there at the beginning?</p> <ul style="list-style-type: none"> <li>● Get stuck with a problem, because of being unable to draw the number of people who were in the bus at the beginning.</li> </ul>	

<sup>30</sup> Term “band” means the thick segment which is often used in place of a segment diagram.

	<ul style="list-style-type: none"><li>• Make a drawing with a question mark or an empty box for an unknown number.</li><li>• Establish a formula on the basis of a drawing, and then find a solution.</li></ul>	
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Discussion topic: through the pupils' actual behavior, reflect on "preparing pupils to live in human society" as a goal for a mathematics lesson.